Amicable Numbers Encyclopedia Article

Amicable Numbers

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Amicable Numbers

Two numbers are said to be amicable (i.e., friendly) if each one of them is equal to the sum of the *proper* divisors of the others, i.e., **whole numbers** less than the given numbers that divide the given number with no remainder. For example, 220 has proper divisors 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, and 110. The sum of these divisors is 284. The proper divisors of 284 are 1, 2, 4, 71, and 142. Their sum is 220; so 220 and 284 are amicable. This is the smallest pair of amicable numbers.

The discovery of amicable numbers is attributed to the neo-Pythagorean philosopher Iamblichus (c. 250- 330), who credited **Pythagoras** with the original knowledge of their nature. The Pythagoreans believed that amicable numbers, like all special numbers, had a profound cosmic significance. A biblical reference (a gift of 220 goats from Jacob to Esau, Genesis 23: 14) is thought by some to indicate an earlier knowledge of amicable numbers.

No pairs of amicable numbers other than 220 and 284 were discovered by European mathematicians until 1636, when the French mathematician **Pierre de Fermat** (1601-1665) found the pair 18,496 and 17,296. A century later, the Swiss mathematician **Leonhard Euler** (1707-1783) made an extensive search and found about 60 additional pairs. Surprisingly, however, he overlooked the smallest pair, 1184 and 1210, which was subsequently discovered in 1866 by a 16-year-old boy, Nicolo Paganini.

During the medieval period, Arabian mathematicians preserved and developed the mathematical knowledge of the ancient Greeks. For example, The polymath Thabit ibn Qurra (836-901) formulated an ingenious rule for generating amicable number pairs: Let $a = 3(2^n) - 1$, $b = 3(2^{n-1}) - 1$, and $c = 9(2^{2n-1}) - 1$; then, if a, b, and c are primes, 2^nab and 2^nc are amicable. This rule produces 220 and 284 when n is 2. When n is 3, c is not a prime, and the resulting numbers are not amicable. For n = 4, it produces Fermat's pair, 17,296 and 18,416, skipping over Paganini's pair and others.

Amicable numbers serve no practical purpose, but professionals and amateurs alike have for centuries enjoyed seeking them and exploring their properties.