Bessel Function Encyclopedia Article

Bessel Function

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Bessel Function

Bessel **functions** are solutions to Bessel's differential equation. The Bessel differential equation follows the form $x^2y'' + xy' + (x^2 - n^2)y = 0$, where the prime notation is indicative of a derivative of y with respect to x. This equation results in an essential singularity at x = 0.

The Bessel functions are usually used in relation with the Bessel differential equation, but they also may be obtained from a generating function. If n is an integer, the Bessel function J_n can be obtained from the sum over s from 0 to **infinity** of the quantity $(x/2)^{n+2s}$ $(-1)^s/s!(n + s)!$. This form is allows numerical evaluation of the Bessel functions, although a series solution of the differential equation (such as Frobenius's method) would allow for the same numerical inspection. The generating function can be manipulated to form several different recursion relations amongst Bessel functions. One of the most commonly used recursions is $J_{n-1}(x) - J_{n+1}(x) = 2J'_n(x)$. Another relation which does not involve derivatives is $J_{n-1}(x) + J_{n+1}(x) = (2n/x)J_n(x)$. These relations, combined with the orthogonality relations, are the basis of many useful proofs and manipulations of the Bessel **equations**; the differential equation itself and the generating function are almost never used directly in practical applications.

The Bessel differential equation has two **sets** of solutions, each of which forms an orthogonal set of functions (or **Hilbert space**). The Bessel functions, discussed above and denoted by J_n, are one of those sets of solutions. The other set is the Neumann functions, usually indicated with $N_n(x)$. (These are also called Bessel functions of the second kind.) The boundary or initial conditions of the specific case determine which set of functions are used. The major difference between Bessel and Neumann function is that Bessel functions are finite at **zero**, whereas Neumann functions approach negative infinity. Bessel and Neumann functions may be combined algebraically to form Hankel functions, which also solve the **differential equations** and fit some physical parameters better than either Bessel or Neumann functions would separately.

These functions are commonly used to solve problems with spherical **symmetry**. For example, the vibrations of a tympani drum when struck with a mallet can be expressed as Bessel functions of one type or another. The constants of the equation can be manipulated to fit a wide variety of physical conditions; the important common factor is spherical symmetry of the situation.