Cramer's Rule Encyclopedia Article

Cramer's Rule

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Cramer's Rule

Cramer's rule gives a formula for the entries of a column vector x that satisfies Ax = b, where A is a n x n **matrix** with nonzero determinant and b is a column vector with n entries. A column vector in n x 1 matrix. Let $A_j(b)$ denote the n x n matrix which has the same entries as A except for the jth column which is equal to b. Cramer's rule is that the jth entry of x must be equal to the $\det(A_j(b))/\det A$. The notation det(M) means the determinant of the matrix M.

Here is the **proof** of Cramer's rule. Let e_j be the length n column vector that has jth entry equal to one and all other entries equal to **zero**. The identity matrix is $[e_1\ e_2\ ...\ e_n] = I$. $AI_j(x) = [Ae_1\ ...\ Ax\ ...\ Ae_n] = A_j(b)$. Because the determinant of a product of matrices is equal to the product of their **determinants**, $\det(A)*\det(I_j(x))=\det(A_j(b))$. Using cofactor expansion, it is easy to see that $\det(I_j(x))$ = the jth entry of x. Therefore the jth entry of x = $\det(A_j(b))/\det(A)$.

Cramer's rule is not very practical as an **algorithm** for solving the equation Ax = b because too many computations are involved. Generally, however, there is some margin of error in figuring b. So, it is often useful to know all the solutions for Ax = b as b varies over its margin of error. In this case, Cramer's rule is helpful for understanding the set of solutions.