Lusin's Theorem Encyclopedia Article

Lusin's Theorem

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Lusin's Theorem

Lusin's theorem is an important technical result in measure theory. Let [,] be a closed, nonempty **interval** of **real numbers**. Then let f: [,] $\Re$ be a Lebesgue measurable, real valued **function** defined at each point of the interval, (see the article on the **Lebesgue integral**). In general a function that is a Lebesgue **measurable** can be very complicated and, for example, it can be discontinuous at each point of the interval. However, it is often useful to know that f is in some sense not too far from being continuous. Lusin's theorem provides information of this sort. The precise statement of Lusin's theorem is as follows. Given the Lebesgue measurable function f and a positive number ε, there exists a continuous function g: [,] $\Re$ such that the Lebesgue measure of the set {x ∈ [,]: f(x) g(x)} is less than ε.

There is also a more general version of Lusin's theorem that applies to *regular* measures defined on the **Borel** subsets of a locally compact **Hausdorff** space X, (see the article on **topology**). In this setting a measure on the -algebra of Borel sets in X is said to be regular if it satisfies the following conditions:

- (1) (C) < for all compact subsets C in X,
- (2) for all Borel sets B in X, (B) = inf{(O): B⊆ O and O is open},
- (3) for all open sets O in X, (O) = sup{(C): C⊆ O and C is compact}.

Now suppose that X is a locally compact Hausdorff space and is a regular measure defined on the Borel subsets of X. Then Lusin's theorem shows that in a certain sense a measurable function can be approximated by a continuous function. The statement of Lusin's theorem in this setting is as follows. Let f:X **C** be a Borel measurable, complex valued function, let B be a Borel set in X such that (B) < and f (x) = 0 for all x not in B. Then for every ε > 0 there exists a continuous function g: X > **C** having compact support such that ({x ∈ X: f(x) g(x)} is less than ε. Moreover, the function g can be selected so that

sup{|g(x)|:x∈ X} sup{| f(x) |:x∈ X}.