Power Series Encyclopedia Article

Power Series

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Power Series

A power series expresses a function *f*(*x*) as an infinite sum (or "series") of terms involving **powers** of the **variable** *x*. Power series can be considered a natural extension of **polynomials**, which are finite sums of such terms. All of the important **functions** of **calculus** can be represented by power series; for example:

$e^x = 1 + xx + xx^2/2! + x^3/6! +...$

$\sin(xx) = x - x^3/3! + x^5 5/5! -...$

In addition, unknown functions, such as solutions to **differential equations**, frequently can be represented in this way as well.

When truncated after a finite number of terms, power series usually provide excellent approximations to the functions they represent, and they can easily be computed by hand. For many applications in physics and other sciences, it is easier to work with a "first-order approximation" or "second-order approximation" to a function (in other words, to stop after the *x* or x^2 term in the power series) than to use the function itself. The errors inherent in this approximation may be smaller than other sources of experimental error, and in any case they can be well controlled.

The major practical limitation of power series is that they do not always converge to any real number. Without some guarantee of **convergence**, it is impossible to predict whether a higher-order approximation is really better than a lower-order one.

The usefulness of power series for theoretical mathematics depends on the fact that they can be differentiated and integrated in the same way that ordinary polynomials can. In the early years of calculus, mathematicians tended to accept this on faith, but in the nineteenth century they gradually realized that such statements are valid only under certain conditions about convergence. The drive to understand these conditions was a major force behind the evolution of calculus into the modern subjects of real and complex **analysis**.